

**CURRENT DIRECTIONS IN STRING THEORY AND ABSTRACT
SUPERSYMMETRY**

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I discuss some current thoughts on how low-energy measurements and consistency constrain theories at high energies with emphasis on string theory. I also discuss some recent work on the dynamics of supersymmetric gauge theories.

1. Introduction

In her talk at this meeting Angela Olinto used both broad and fine brush strokes to paint her view of cosmology in the next millennium. In this talk I intend to use only the broadest of brushes, perhaps spray painting would be a better analogy, to discuss our ability to probe Planck scale physics using only low-energy experiments and self-consistency. I will also discuss in very general terms some exciting recent results on the dynamics of supersymmetric gauge theories.

There is a great deal of doom and gloom in particle physics at the moment as a result of the cancellation of the SSC and the meager job market for young particle physicists. There is also great frustration at the success of the Standard Model and our seeming inability to probe what lies beyond it. This has led to statements in the popular press implying that particle physics is approaching a dead end as a field of scientific inquiry. While there is certainly cause for concern, I think such pronouncements are short-sighted and seriously underestimate the ingenuity of both theorists and experimentalists over the long term. To counter this I would like to discuss some reasons for optimism and some new developments which show that our bag of tricks is not yet exhausted.

One valid concern about current particle theory is the vast chasm which has opened up between experiment and some of the theoretical frontiers, especially string theory. There is a tendency in many quarters to regard string theory as a failure, not because of any internal problems, but because it only addresses physics at the Planck scale and hence is inherently untestable in the foreseeable future. While direct tests are certainly hard to imagine, I would like to discuss a number of indirect ways in which physics at the Planck scale and particularly string theory are already playing a significant role in our view of low-energy physics.

Before beginning let me mention a remark which I have heard with varying attributions to the effect that the problem is not that we take our theories too seriously, but rather that we do not take them seriously enough. Examples abound, ranging from black holes to quarks to gauge theories and electroweak theory. In each case the theory

was initially viewed as either a mathematical curiosity or abstraction or as a toy model. In each case the theory was more real than physicists could bring themselves to believe. As a mature field, particle physics is more tightly constrained than newer fields and often a small amount of data is sufficient to develop a very rigid theory. Electroweak theory is a particularly good example of this. I would like to encourage you to take string theory as seriously as I think it deserves to be taken, not as a mathematical abstraction or a toy model of quantum gravity, but rather as a real theory of the world that may shed light on real phenomena.

2. Large-scale small-scale connections in particle physics

2.1. The Standard Model

We are currently both blessed and cursed by the incredible success of the Standard Model as reviewed at this meeting by Jon Rosner.¹ In spite of its success, it is certainly true that it is not a final theory of everything, but rather only a low-energy effective theory of something, namely the light particles which include the three generations of quarks and leptons, the gauge bosons of $G = SU(3) \times SU(2) \times U(1)$, and presumably one or more Higgs bosons. There are many reasons to believe that the Standard Model is only a stepping stone on the way to a more complete theory. First of all, it does not include gravity. Second, it almost certainly does not exist formally as a quantum field theory due to the lack of asymptotic freedom of some of the coupling constants. Finally, it has too many loose ends, requiring the specification of some 20 odd parameters. One of the main goals of particle physics at the end of the 20th century is to figure out what lies beyond the Standard Model.

Since we do not believe the Standard Model is a final, self-contained theory, we should view it only as a low-energy effective theory that describes physics in an approximate (but nonetheless remarkable accurate) way at low-energies. According to the philosophy of effective field theory² if the Standard Model is effective below a scale Λ then in the effective Lagrangian we should write down all interactions involving the light degrees of freedom which are compatible with the symmetries of the problem. Furthermore, all dimensionfull couplings should be given by appropriate powers of Λ up to factors which are (roughly) of order one. Very schematically this gives

$$\mathcal{L}_{eff} = (\Lambda^4 + \Lambda^2 \phi^2) + ((D\phi)^2 + \bar{\psi} \not{D} \psi + F_{\mu\nu} F^{\mu\nu} + \phi \bar{\psi} \psi + \phi^4) + (\frac{\bar{\psi} \psi \bar{\psi} \psi}{\Lambda^2} + \dots) \quad (1)$$

in a notation where ϕ stands for generic scalar field, ψ for generic fermion fields, and $F_{\mu\nu}$ for the gauge fields. The first set of terms have positive powers of Λ , they are the most relevant terms in understanding the low-energy theory. They consist of the cosmological constant term and a mass term for any scalar particles. Note that a mass term for fermions is forbidden in the Standard Model by gauge invariance. The second set of terms are independent of Λ (or have at most log dependence on Λ) and consist of all the interactions of the usual Standard Model except for the Higgs mass term. The final set of terms is really an infinite expansion which includes all terms with negative powers of Λ . These terms are non-renormalizable in the usual sense, and are irrelevant in very low-energy processes, but are nonetheless there in a generic low-energy effective field theory.

Now one beautiful thing about the standard model is that *all* the renormalizable terms which are consistent with the gauge symmetries are present. It is true that some of the couplings are small for poorly understood reasons (e.g. the electron and up and down quark Yukawa couplings), but we are not forced to set any couplings to zero in order to agree with experiment (except perhaps the cosmological constant) .

The natural question to ask is, what is the value of Λ ? Historically the first answer was that $\Lambda = \Lambda_{GUT} \sim 10^{15} GeV$ based on unification of the gauge couplings as well as on the absence of proton decay, flavor changing neutral currents, and other non-standard model processes. The point being that such effects are generally contained in the non-renormalizable interactions but are sufficiently small if suppressed by powers of Λ_{GUT} . This answer leads to two great embarrassments which involve the two relevant terms in \mathcal{L}_{eff} . The first is the cosmological constant problem. The value of the cosmological constant is *much* less than Λ_{GUT}^4 for reasons that are completely mysterious. The relevance of this term is clear. If the cosmological constant were of order Λ_{GUT}^4 the universe would have come and gone in a Planck time without any physicists to worry about the problem. The second embarrassment is usually called the hierarchy problem, namely that the Higgs mass is not Λ_{GUT}^2 but must be more of order M_W^2 . There is of yet no good answer to the first problem, but at least we can appeal to an ignorance of quantum gravity. The second problem is a problem of particle physics and must be faced up to.

If we are not allowed unnatural fine tuning then the only possibility is that Λ is not Λ_{GUT} but rather is not far from the weak scale, say generically $\Lambda \sim 1TeV$. There are currently two main speculations as to the new physics at this scale. The first is that there are new strong interactions, that is technicolor, top quark bound states or whatever; the second is that this is the scale of new supersymmetric particles. Whatever the new physics, we know that it must have special features. In particular, we know that many of the non-renormalizable interactions in \mathcal{L}_{eff} cannot be present if they are suppressed only by powers of $\Lambda \sim 1TeV$. Of course this is not a surprise. It is well known that technicolor and supersymmetric models are strongly constrained by the demands that proton decay, CP violation and flavor changing neutral currents be compatible with present experimental limits. But it is worth emphasizing that this is a positive feature. It tells us that the new dynamics at the TeV scale is not generic but must have special structures or symmetries which forbid these effects.

2.2. The MSSM

Currently the most popular proposal for going beyond the standard model is based on supersymmetry. There is now a standard minimal model incorporating supersymmetry, the MSSM.³ We should first understand how supersymmetry solves the naturalness problem. In the early days of supersymmetry one sometimes heard the following idea. Supersymmetry if unbroken requires the equality of fermion and boson masses. Since fermions are required to be massless in the Standard Model before weak symmetry breaking, bosons would also have to be massless before electroweak breaking. A Higgs mass of order the weak scale would then be generated provided that supersymmetry was effectively broken near the weak scale.

This idea does not work in the most naive sense. The reason is as follows. In

$N = 1$ supersymmetry chiral fermions are paired with complex scalars. Thus one standard Higgs doublet is paired with a chiral fermion doublet. Now adding such a multiplet to the Standard Model is not possible without upsetting the delicate cancellation of anomalies between chiral fermions. So what is done is to add two multiplets corresponding to two Higgs doublets H_1, H_2 with opposite hypercharge. Their fermion partners are then non-chiral and there is no problem with anomalies. But then there is a supersymmetric and gauge invariant mass term allowed for the multiplets which in a supersymmetric notation takes the form

$$W = \mu H_1 H_2. \quad (2)$$

Since this term is supersymmetric and gauge invariant there is no reason for it not to be of order the Planck scale or whatever scale comes beyond the MSSM and thus the hierarchy or naturalness problem is not really solved. In the context of the MSSM this is usually called the “ μ problem”.

It is tempting to try to forbid this term by some symmetry, but this also leads to problems, in particular with $\mu = 0$ the MSSM has a standard Peccei-Quinn axion which is ruled out experimentally. So we need μ to be non-zero and of order the weak scale. This suggests that the value of μ should be connected with supersymmetry breaking, even though it is by itself supersymmetric.

There are in fact solutions to the μ problem in supergravity and string theory where this is precisely what happens. μ is zero initially for reasons that have to do with the precise high-energy structure of the theory and a non-zero value is induced only through supersymmetry breaking effects.⁶ Thus at least in one class of models the consistency of the MSSM requires the existence of special features at the Planck scale.

It was argued earlier that the theory beyond the standard model would have to have special features in order to ensure that baryon number violation and flavor changing neutral currents (FCNC) are sufficiently small. In the MSSM one can say quite specifically what is required. The presence of scalar partners of quarks (squarks) means that there are renormalizable couplings which violate baryon number. These must be very small to be phenomenologically acceptable. The simplest solution is to use a discrete symmetry to force them to be zero. This is conventionally done by defining a Z_2 discrete symmetry called R -parity which has eigenvalue $(-1)^{3(B-L)+2S}$ when acting on a state of baryon number B , lepton number L and spin S . Equivalently, it is $+1$ on all standard model particles and -1 on all their superpartners. R parity also forbids all renormalizable couplings which violate lepton number. R parity also implies that the lightest supersymmetric partner (LSP) is absolutely stable since there is nothing it can decay into while preserving R -parity. Thus R -parity has very important phenomenological consequences. There are other discrete symmetries which do not forbid all these couplings but which nonetheless are phenomenologically acceptable. Perhaps the most attractive is a Z_3 matter parity discussed by Ibanez and Ross.⁴ The central position played by such discrete symmetries makes it important to ask whether we really expect exact discrete symmetries to exist.

Global discrete symmetries may be approximate symmetries of low-energy effective Lagrangians, but it is difficult to see why they should not be violated at sufficiently high energies, say by gravitational effects. However discrete symmetries can be exact

if they are gauge symmetries. The distinction between gauge and global symmetries applies whether the symmetry is continuous or discrete. Global symmetries are true symmetries of the configuration space of a theory while gauge symmetries are only a redundancy in our description of the theory or equivalently of its configuration space. This is why one does not find Nambu-Goldstone bosons in “spontaneously broken” gauge theories. Gauge symmetries cannot be spontaneously broken since they are just a redundancy of our description, equivalently there are no flat directions in the configuration space due to gauge symmetries which could give rise to massless modes.

With this in mind it should be clear that discrete symmetries can also be classified in the same way. One simple way such symmetries can arise is by “breaking” of a $U(1)$ gauge symmetry down to a Z_N subgroup. The low-energy theory will have a Z_N discrete symmetry which is a gauge symmetry since it is a subgroup of the original $U(1)$ gauge symmetry. Such discrete gauge symmetries can arise by other means and in fact they are very common in compactifications of string theory. There are also consistency conditions for discrete gauge symmetries coming from a discrete version of anomaly cancellation,⁵ these conditions are satisfied both for the usual Z_2 R-parity and for the Z_3 matter parity. Thus we again find that a phenomenological requirement on the MSSM leads us to conclude that it should be imbedded in a theory at high energies which can contain discrete gauge symmetries.

Probably the outstanding problem in the MSSM is understanding the mechanism responsible for supersymmetry breaking. Ultimately the hierarchy problem must be solved by explaining why the effective scale of supersymmetry breaking is near the weak scale. Also, without an understanding of supersymmetry breaking we can only parametrize the breaking by soft terms in the low-energy Lagrangian. Although only four such terms are often used based on certain “minimal” assumptions, these assumptions are totally unjustified from a theoretical point of view and there are really 60 odd parameters needed to specify the most general soft breaking terms. As a result the model has little predictive power. At present we don’t know whether supersymmetry is broken by the dynamics of some hidden strongly coupled gauge sector, by non-perturbative effects in the visible sector, or by some intrinsically stringy mechanism. Up until recently the most popular models have been hidden sector models where supersymmetry is actually broken at a rather large scale, but only communicated to the visible world by gravitational effects, leading to an effective scale of supersymmetry breaking around the TeV scale.

Thus in the MSSM there are many hints as to the structure of whatever theory lies beyond the MSSM, perhaps at the Planck scale. It must solve the μ problem, allow for discrete symmetries, preferably gauged, and perhaps provide a mechanism for dynamical supersymmetry breaking. Such a mechanism is further constrained by the allowed values of the soft supersymmetry breaking parameters.

2.3. *String Theory*

In the previous two sections I have tried to argue that the low-energy structure we have observed and hope to observe in the future carries important clues about the behavior of physics at very short distances, e.g. the Planck scale. Currently the most successful model of Planck scale physics is superstring theory. It is not my intention

to review string theory or even current progress in string theory in this talk. Instead I would just like to mention a rather simple but important way in which string theory itself provides a connection between physics on vastly different scales.

This comes about by asking the question “How big is a string?”. Consider for simplicity a closed bosonic string which traces out a closed loop in space parametrized by a coordinate σ with $0 < \sigma \leq 2\pi$. We can write a normal mode expansion for the coordinate of the string as

$$X(\sigma) = x_{cm} + \sum_n \left(\frac{x_n}{n} e^{in\sigma} + \frac{\tilde{x}_n}{n} e^{-in\sigma} \right) \quad (3)$$

with x_{cm} the center of mass of the string and x_n , \tilde{x}_n being the Fourier coefficients. When time dependence is added the x_n correspond to excitations running around the string counterclockwise and the \tilde{x}_n to excitations running around the string clockwise.

One measure of the size of the string is the average deviation of any point on the string from the location of the center of mass, that is

$$R^2 = \langle (X(\sigma) - x_{cm})^2 \rangle \quad (4)$$

We want to calculate this average quantum mechanically in which case the x_n and \tilde{x}_n become harmonic oscillator creation and annihilation operators for $n < 0$ or $n > 0$ respectively. Clearly for a very excited string R will be very large, but what is rather strange is that due to zero-point fluctuations R is large even in the string ground state. In the string ground state one easily finds that the n^{th} oscillator contributes a factor of $1/n$ and

$$R^2 \sim \sum_n \frac{1}{n} \quad (5)$$

To make sense of this divergent sum we have to understand what it means physically to measure the size of a string. Any real measuring device will have some finite time resolution τ_{res} . On the other hand the divergence comes from modes with large n and these modes have frequencies which grow as n . Since a real measuring device cannot measure frequencies greater than $1/\tau_{res}$ it makes sense to cut off the sum at a mode number corresponding to the time resolution of the measuring device. Doing this somewhat more carefully than described here and reinstating dimensions gives⁷

$$R^2 \sim l_s^2 \log(P_{tot}/\tau_{res}) \quad (6)$$

where l_s is the string scale which is of order the Planck scale (it is less than the Planck scale by a power of the dimensionless string coupling constant) and P_{tot} is the total transverse momentum of the string.

This equation is quite remarkable when one thinks about it. It says that the size of a string depends on how fast you can measure it, and that the faster you can measure it, the bigger it appears. We are used to the idea, which follows from the uncertainty principle, that we need high energies (short times) to probe short distance scales. But as we approach the Planck scale things change, at least in string theory. We need high energies (short times) to probe large distance scales, or at least the large size behavior of strings.

Considerations like this lead to a “string uncertainty principle”⁹ which relates the uncertainty in the size of a string Δx to its energy E : (in units with $\hbar = c = 1$)

$$\Delta x \geq \frac{1}{E} + \frac{E}{M_{Pl}^2}. \quad (7)$$

This equation incorporates both the usual quantum mechanical uncertainty and the less well understood uncertainty due to the growth of string states at high energies.

This behavior is responsible for many qualitative features of string theory. For example it is generally believed that string theory has no ultraviolet divergences and this is certainly backed up by many explicit calculations. The above behavior explains why. At very high energies strings become very large and floppy and there is no large concentration of energy at a point which could lead to bad high-energy behavior. It also sounds temptingly like what one wants in order to solve the cosmological constant problem. There one needs some peculiar connection between large scale physics (as governed by the smallness of the cosmological constant which explains the largeness of the present universe) and small scale physics (which should govern the leading contributions to the cosmological constant.) However I do not know of any serious attempt to solve this problem in string theory which utilizes this feature of string theory.

2.4. *Black Holes*

Over the last few years there has been a resurgence of interest in the quantum mechanics of black holes, inspired in part by the hope that string theory or tools derived from string theory may help to solve some of the outstanding puzzles. The basic outstanding questions are

1. What is the correct treatment of the black hole singularity? Is the breakdown of classical general relativity a sign that quantum gravity is important, or that new short distance classical effects are present (e.g. classical string theory)?
2. Is the Hawking evaporation of black holes consistent with the formalism of quantum mechanics? In particular, can pure states evolve to mixed states due to black hole intermediate states?
3. Is there a microscopic description in terms of state counting of the Bekenstein-Hawking black hole entropy $S = A/4$ with A the area of the event horizon?

In spite of much activity, there is as yet no consensus on the answers to these questions. If black hole entropy is associated with new states on the horizon of a black hole then these states must either show up in a more careful treatment of quantum gravity in the presence of black holes or they must be associated to new states in a more complete theory of quantum gravity such as string theory. The first point of view has been explored recently in the context of toy models in $2+1$ dimensions by Carlip, Teitelboim and collaborators.⁸ The second point of view has been explored by Susskind and collaborators¹⁰ and fits in nicely with the behavior of string theory discussed earlier and with the general idea of looking for connections between short and long distance physics.

Following Susskind and collaborators consider the difference between a point particle and a fundamental string falling into a black hole as seen by an outside observer. In either case, the outside observer sees any radiation or signal coming from the infalling object redshift exponentially as it approaches the horizon. Thus a measurement made at a fixed frequency probes the structure of the infalling object on increasingly shorter time scales. According to the previous description this should mean that the outside observer in effect “sees” a string increase in size as it falls into a black hole. This suggests that as it reaches the horizon the string is effectively smeared over the whole horizon of the black hole. It seems plausible that such smeared string states could account for the entropy of the black hole, but there has so far been no explicit calculation which shows this to be the case. Also, the word “see” should be taken with a grain of salt. Because of the exponential redshift in the the radiation emitted by an object falling into a black hole, it becomes harder and harder to actually see the physics of the string spreading. Presumably it eventually becomes mixed up with the Hawking radiation in some complicated way.

The question of loss of coherence or information in black hole formation and evaporation should in some sense be tied to the microscopic description of entropy. If it is possible to account for the states and their thermal behavior in an honest way, then we would expect that there would be no information loss once the detailed interaction with the horizon states is take into account. On the other hand, the whole notion of new states on the horizon is rather peculiar and contrary to many of the developments in general relativity over the last twenty years. In particular, to an infalling observer there is absolutely nothing special about the horizon and no reason to expect any new states, nor anyplace they would be expected to be. Thus there would have to be some very strange new kind of complementarity between the description of physics by different observers.

There have also been many attempt to resolve these problems in toy models based on $1 + 1$ dimensional versions of gravity, but again there seems to be no consensus. An up to date overview of this problem can now be found on the Web.¹¹

2.5. *Inflation*

I will be very brief here since Inflation has already been discussed at this conference in the lecture by Angela Olinto.¹² There are several reasons why inflation provides a particularly compelling example of relations between small scale and large scale physics. First of all, there is so far no microscopically compelling model of inflation, that is a natural particle physics model which gives rise to an inflaton field and potential that leads to the proper amount of inflation and the density perturbations of the required amplitude. Such a model would provide a direct link between physics near the GUT scale and the size of the current universe. More spectacularly, in inflation models the density perturbations which eventually grow into the stuff we see about us are supposed to have their origin in the quantum mechanical fluctuations of the inflaton field, what more dramatic connection between microscopic and macroscopic physics could we hope for? Finally, our current understanding of inflation rests on the idea that there was a past epoch of the universe where the vacuum energy density, a.k.a. the cosmological constant was non-zero. While this seems inescapable in many models

given that the present cosmological constant is zero, one cannot escape an uneasy feeling that our whole picture of inflation may change dramatically if we ever understand the cosmological constant problem.

3. Dynamics of Supersymmetric Gauge Theories

3.1. Motivation

During the last 15 years two theories have been developed in some detail that involve supersymmetry in an essential way. The first is superstring theory. Superstring theory allows us for the first time to address the physics of the Planck scale in a well defined way. It is also well understood by now that there are many solutions to superstring theory that leave us with an effective four-dimensional supersymmetric theory which resembles the standard model in the sense that it can contain some number of chiral fermion generations in representations of a gauge group which is usually somewhat larger than the gauge group of the standard model. On the other hand there is so far no understanding of which (if any) of these vacua is picked out dynamically as the true vacuum (they are equally good in perturbation theory). There may also be many inequivalent vacua left after all dynamical effects are included in which case one needs some further principle (e.g. quantum cosmology) in order to make contact with our particular world. Another aspect of this problem is that most superstring vacua contain many massless scalar fields called moduli which have no potential. Different vacuum expectation values for these moduli correspond to different choices of superstring vacua. The “moduli problem” in superstring theory is the problem of how a suitable potential is chosen by the dynamics for these fields.

The second theory where supersymmetry has played a crucial role is of course the supersymmetric extension of the standard model (MSSM) discussed earlier. There the main roadblock to detailed predictions is our lack of understanding of supersymmetry breaking.

One of the main themes in particle theory over the last few years has been the attempt to tie together the structure needed in the MSSM with the sort of structures that arise in superstring theory. This not only provides further constraints on the MSSM but also may shed some light on both the moduli problem and the problem of supersymmetry breaking.

Over the last year there has been dramatic progress, building on work in the early 1980’s, in understanding the dynamics of supersymmetric gauge theories in four dimensions. Although this work has not yet answered either of the above questions, it has introduced new techniques, and also promises to shed light on some old problems in non-supersymmetric gauge theories. As a result I would like to give a brief discussion of these new developments following a recent paper by Seiberg and Witten.¹³

3.2. $N = 1$ Supersymmetry

Before discussing the results of Seiberg and Witten it will be useful to very briefly review a few facts about theories with $N = 1$ supersymmetry. These theories are the basis for supersymmetric extensions of the standard model (the MSSM) because only theories with $N = 1$ supersymmetry are compatible with having chiral fermions. Lagrangians with $N = 1$ supersymmetry are most easily constructed in terms of super-

fields in superspace. The coordinates of superspace consists of the usual spacetime coordinates x^μ and fermionic coordinates θ which can be thought of as the two complex components of a Weyl fermion. The basic matter multiplet, consisting of a Weyl fermion and a complex boson can be packaged into a complex scalar function on superspace, $\Phi(x, \theta)$ called a chiral superfield. The general Lagrangian then has two types of terms, written as

$$\mathcal{L} = \left(\int d^2\theta d^2\bar{\theta} K(\Phi, \Phi^*) \right) + \left(\int d^2\theta W(\Phi) + h.c. \right) \quad (8)$$

and referred to as D terms and F terms respectively. For a renormalizable Lagrangian the D terms contain the kinetic energy terms while the F terms contain potential terms and Yukawa interactions. The important point to note is that the F terms are determined purely by functions of Φ while the D terms involve functions of both Φ and Φ^* . This is peculiar to supersymmetry and has no analog in ordinary non-supersymmetric field theories. It has been known some time that F terms are not renormalized in perturbation theory. The proofs of this rely on the detailed structure of superspace perturbation theory. More recently it has been realized that there is a very simple and powerful argument for this non-renormalization which can also be extended to prove results about non-perturbative effects.

The argument is best illustrated with a simple example.¹⁴ For a single scalar superfield the most general renormalizable choice of W is

$$W(\Phi) = \frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3 \quad (9)$$

When $m = \lambda = 0$ this theory would have two $U(1)$ symmetries, consisting of changing the phase of the scalar and fermion components of Φ separately. The idea is to view the parameters m and λ as expectation values of chiral superfields and these two symmetries as being spontaneously broken. Thus we replace

$$m \rightarrow \langle \Phi_m \rangle \quad (10)$$

$$\lambda \rightarrow \langle \Phi_\lambda \rangle \quad (11)$$

and find that the theory is invariant under the two $U(1)$ symmetries

$$S : \quad \Phi_m \rightarrow e^{-2i\alpha} \Phi_m \quad (12)$$

$$\Phi_\lambda \rightarrow e^{-3i\alpha} \Phi_\lambda \quad (13)$$

$$\Phi \rightarrow e^{i\alpha} \Phi \quad (14)$$

and

$$R : \quad \Phi_m(\theta, x) \rightarrow \Phi_m(\theta, x) \quad (15)$$

$$\Phi_\lambda(\theta, x) \rightarrow e^{-i\beta} \Phi_\lambda(e^{i\beta}\theta, x) \quad (16)$$

$$\Phi(\theta, x) \rightarrow e^{i\beta} \Phi(e^{-i\beta}\theta, x) \quad (17)$$

Now since we can view the symmetry as being spontaneously broken, we know that it must be respected by the one-particle irreducible effective action. But the only terms

allowed in the 1PI effective action by these two symmetries are the original terms! Thus there can be no renormalization of the potential terms. It should be mentioned that there are some subtleties with this argument when there are massless particles.¹⁵ Similar arguments can be used to determine various non-perturbative effects in these theories, but only through the contribution to F terms. The argument does not work for D terms since they depend on both Φ and Φ^* . If there were an additional symmetry relating D and F terms then one would have very strong constraints on the dynamics of the theory. This is precisely what happens in theories with $N = 2$ supersymmetry.

3.3. $N = 2$ Supersymmetry

The simplest Yang-Mills theory with $N = 2$ supersymmetry contains a single supermultiplet of fields consisting of what we might call a gauge boson and gaugino plus a Higgs boson and Higgsino. These fields are all in the adjoint representation of the gauge group. Taking the gauge group to be $SU(2)$ they are all isotriplet fields. The Higgs ϕ is a complex field. $N = 2$ supersymmetry dictates that the potential term for ϕ is

$$V(\phi) = \text{Tr}[\phi, \phi^*]^2. \quad (18)$$

Clearly $V = 0$ for any configuration of ϕ which lies in a single direction in group space, say $\phi = a\tau_3/2$ with a an arbitrary complex number. A gauge invariant way of describing this is to say that there is a zero of V for every complex value of $u = \text{Tr}\phi^2 = a^2/2$.

The fact that classically there is a continuous set of vacua specified by u is a feature which is common to many supersymmetric theories. In general it is a disaster for phenomenological applications of supersymmetry. This is both because it reflects the presence of massless scalars in the low-energy theory and because it makes it unclear which vacuum is the right one for doing physics. This latter problem is particularly onerous in low-energy supersymmetric string theory. The beauty of the recent ideas is to take this bad feature of supersymmetric theories and to make use of it to study the dynamical structure of these theories.

To see how this works first consider this theory at some non-zero value of u . We then have $SU(2)$ broken down to a $U(1)$ which I will call electromagnetism. The spectrum of the theory consists of a massless photon supermultiplet, massive W^\pm supermultiplets, and also massive monopole supermultiplets. At very low-energies the theory just looks like a supersymmetric form of electromagnetism with some electric coupling e_{eff} . I first want to explain how e_{eff} is related to the value of u . This theory is asymptotically free at high energies, that is $\beta_e < 0$ at scales above the masses of all particles. As we drop below the masses of charged particles they decouple from loops and the coupling constant stops running. Clearly the asymptotic value of the coupling constant at low energies depends on the scale of masses in the theory. These masses are determined by the value of u . So, specifying a value of u is equivalent to specifying the coupling e_{eff} in the low-energy theory.

As long as we only consider the massive electrically charge particles and the photon then the low-energy physics is completely specified by e_{eff} . When we include the monopoles as well it is necessary to also specify the effective value of the θ angle, θ_{eff} . This parameter is the electromagnetic analog of the θ angle in QCD . In QED it also leads to CP violation by giving magnetic monopoles of magnetic charge n_M an

electric charge given by

$$Q_{el} = \frac{n_M}{2\pi} \theta_{eff} \quad (19)$$

The upshot of this is that the complex parameter u in fact determines two real low-energy parameters (e_{eff}, θ_{eff}) or equivalently, one complex low-energy parameter.

Now we would like to know what happens to the theory and the spectrum of states as we move around between classical vacua labelled by u , or equivalently move in the space of couplings (e_{eff}, θ_{eff}) that govern the low-energy field theory. First of all it can be shown fairly easily that at weak coupling $\theta_{eff} \propto \text{Im} u$. Thus at weak coupling the effect of shifting the imaginary part of u is simply to shift the electric charge of magnetic monopole states. Clearly it would be much more interesting to determine what happens when g_{eff} changes since this could tell us about the behavior of the theory at strong coupling.

Unfortunately the analysis get quite a bit more complicated and I cannot do it justice in this talk. Let me just mention the following points.

1. The one-loop beta function predicts that the coupling e^2 should blow up in the infrared and then become negative. Clearly this is physically unacceptable. Of course there is no reason to believe the one-loop result at strong coupling.
2. The possible behavior of the theory as a function of u is highly constrained by the $N = 2$ supersymmetry. In particular the u plane must be a special kind of complex manifold, a Kahler manifold.
3. By utilizing the supersymmetry and analytic structure one can find a simple guess for the behavior of the theory as a function of u which passes many non-trivial consistency tests.

The resulting structure found by Seiberg and Witten has many remarkable features. First of all, there is a kind of “dual” behavior at strong coupling where as one moves in u the electrically charged states pick up magnetic charge, just as the magnetic states can pick up charge at weak coupling. Second, one finds singular points where monopole states are becoming massless. It is possible to perturb the theory by breaking $N = 2$ supersymmetry to $N = 1$ supersymmetry in such a way that the monopoles mass squared becomes negative, indicating an instability to monopole condensation. It has long been thought that confinement may be described by a “dual” superconductor in which magnetic charges condense. In these models one has for the first time a concrete realization of these ideas.

There seem to be two major areas where these results may have a broad impact. The first is in the construction of new models for dynamical supersymmetry breaking. As I mentioned earlier, the lack of a convincing mechanism for supersymmetry breaking is one of the major obstacles to obtaining predictions from the MSSM. The second area is the dynamics of strongly coupled gauge theories, both supersymmetric and non-supersymmetric. The models discussed by Seiberg and Witten are in a sense toy models, but they contain most of the features of non-trivial gauge theories including running couplings, non-trivial scattering, confinement, and chiral symmetry breaking. In addition the way these effects are realized is rather dramatic involving an infinite

resummation of non-perturbative instanton effects. It seems possible that some of these features may extend beyond these specific models.

4. General Remarks

I would like to end this talk with a few general remarks about the prospects for progress in string theory in the upcoming years.

The string revolution is now ten years old. The discovery of anomaly cancellation by Green and Schwarz was discussed at the November 1984 DPF meeting. Roughly a year later the basics of superstring models of unification were established through Calabi-Yau compactifications of the heterotic string. Since then there has been a great deal of effort and progress in formal areas, but little that an experimentalist would find applicable to current or future experiments. As a result a certain amount of pessimism and scepticism has arisen regarding the future of string theory.

It is worthwhile recalling the situation before string theory. As is the case now, in 1984 there were no startling new experimental results and the leading idea for unification, Kaluza-Klein theory, had run into insurmountable obstacles. However there were a number of novel theoretical ideas and structures floating around which did not fit into any coherent whole. As examples I might mention connections between the renormalization group and geometry, the beautiful structures of two-dimensional current algebras or Kac-Moody algebras, the development of realistic supersymmetric extensions of the standard model with hidden sector supersymmetry breaking, as well as a number of other purely theoretical constructions. Many of these are now regarded as either part of the structure underlying string theory or as possible low-energy consequences of string theory.

If we think of the situation today there are again many new ideas floating around, many are classified in a general way as “string theory” but in fact many have very little to do with string theory in any direct way. For example there are matrix models of two-dimensional string theory, new ideas concerning the structure of black holes with toy models in $1+1$ and $2+1$ dimensions, topological field theory with its connection to many deep mathematical structures, new ideas about the structure and dynamics of supersymmetric gauge theory, a deeper understanding of special two-dimensional systems, etc. Of course this is not to say that another revolution comparable to string theory is around the corner, but rather that there has been continuing theoretical progress and that there are new connections and structures which will undoubtedly fit into a more coherent picture in the future.

I have tried to take an optimistic, but I hope not unreasonable point of view in this talk. My point of view is that in spite of the dearth of new experimental results, there are new interesting theoretical ideas and tools and that these plus consistency and constraints from the Standard Model provide us with non-trivial information about the structure of string theory or whatever theory governs physics at the Planck scale. I am certainly not suggesting that we can find the theory of everything without additional experimental input. I am suggesting that as our theories become more tightly constrained each additional piece of experimental information carries much more weight. I think this holds out hope that we will eventually understand Planck scale physics without building a Planck scale accelerator.

5. Acknowledgements

I would like to thank the organizers for the invitation to speak and the other speakers for their lively and interesting presentations.

References

1. J. Rosner, plenary talk, this conference.
2. For a review see J. Polchinski, “Effective Field Theory and the Fermi Surface” in *Recent Directions in Particle Theory* (Proceedings of the 1992 Theoretical Advanced Study Institute in elementary Particle Physics), edited by J. Harvey and J. Polchinski (World Scientific, Singapore, 1993).
3. H. Haber, “Introductory Low-Energy Supersymmetry” in *Recent Directions in Particle Theory* (Proceedings of the 1992 Theoretical Advanced Study Institute in elementary Particle Physics), edited by J. Harvey and J. Polchinski (World Scientific, Singapore, 1993).
4. L. E. Ibanez and G. G. Ross *Nucl. Phys.* **B368** (1992) 3
5. L. E. Ibanez and G. G. Ross *Phys. Lett. B* **260** (1991) 291; T. Banks and M. Dine *Phys. Rev. D* **45** (1992) 1424
6. J.A. Casas and C. Munoz *Phys. Lett. B* **306** (1993) 288; V. Kaplunovsky and J. Louis *Phys. Lett. B* **306** (1993) 269; G.F. Giudice and A. Masiero *Phys. Lett. B* **206** (1988) 480
7. L. Susskind *Phys. Rev. Lett.* **71** (1993) 2367
8. S.Carlip, “The statistical mechanics of the (2+1)-dimensional black hole,” gr-qc/9409052 and references therein.
9. D. J. Gross and P. F. Mende *Phys. Lett. B* **197** (1987) 129; *Nucl. Phys.* **B303** (1988) 407
10. L. Susskind *Phys. Rev. D* **50** (1994) 2700; L. Susskind, L. Thorlacius and J. Uglum *Phys. Rev. D* **48** (1993) 3743; A. Mezhlumian, A. Peet, and L. Thorlacius, *Phys. Rev. D* **50** (1994) 2725
11. S. Giddings, “Quantum Aspects of Gravity” page in the Virtual Review <http://www.het.brown.edu/physics/review/index.html>.
12. A. Olinto, plenary talk, this conference.
13. N. Seiberg and E. Witten *Nucl. Phys.* **B426** (1994) 19
14. N. Seiberg, hep-th/9408013.
15. M. Dine and Y. Shirman *Phys. Rev. D* **50** (1994) 5389